products involving the out-of-band signals can appear in-band. For example, if $f_o' = f_o/2$, then the second harmonic (a second-order product) has this property. For broad-band signals such as thermal noise, second-order intermodulation products of this type are incoherent with the desired signals and therefore appear as an increase in noise at high signal levels, rather than as a reduction in gain (saturation). However, third-order (and higher odd-order) mixing products can be coherent and can contribute to the saturation. This can be expected to be much less important than the effects noted in the first and second conclusions.

The use of (4) to analyze fully a given junction and embedding network is especially difficult. It is necessary to solve for the voltages and currents at all frequencies simultaneously, given the (linear) constraints imposed by the embedding network. The situation can be simplified by considering only the three-port model, where the embedding network presents a short circuit at all frequencies f_m, f_m' for which $|m|>1$; this leaves nonzero voltages at six frequencies, three in-band and three out-of-band. A further simplification would be to neglect all but first-order mixing products. A solution might then be obtained iteratively by first using the small-signal Y matrix to find the approximate signal voltages, then using these in (4) to estimate the currents, then using the currents in the embedding network to obtain improved approximations to the voltages, and repeating until convergence. This still would not treat the broad-band noise case. It remains a difficult calculation, and the author intends to pursue it in a future publication.

IV. Conclusions

It has been demonstrated by both analysis and intuitive argument that gain saturation in an SIS mixer results when the total signal voltage across the junction becomes too large. It is emphasized that this includes voltages at frequencies outside the bands of interest of the mixer, such as arise when the input is broad-band noise. To obtain the largest dynamic range, the designer must ensure that the embedding network suppresses such voltages. The network can do this by approaching a short circuit at out-of-band frequencies. In high-gain mixers, the largest voltages normally occur at the output frequency (IF); in such cases, a carefully designed IF filter can significantly improve the dynamic range.

Appendix

For an arbitrary time function of applied voltage $V(t) = V_{dc} + V_{ac}(t)$, the expected value of current in a tunnel junction is given by [7]

$$I(t) = 2\,\mathrm{Re}\left\{\int_{-\infty}^{t} I_{FT}(t-t')\, e^{i2\pi(q/h)\int_{t'}^{t} V_{ac}(\tau)\,d\tau}\, dt'\right\} \quad \text{(A1)}$$

where $I_{FT}(t)$ is the Fourier transform of the dc current–voltage characteristic of the junction $I_{dc}(V)$ with respect to transform variable $f = q(V - V_{dc})/h$:

$$I_{FT}(t) = \int_{-\infty}^{\infty} I_{dc}(V_{dc} + hf/q)\, e^{-i2\pi ft}\, df. \quad \text{(A2)}$$

This formula can be easily derived from [6, eqs. (2.8), (2.11), and (2.16)]. Then if $V(t)$ is given by using (3) in (1), (A1) becomes

$$I(t) = 2\,\mathrm{Re}\left\{\int_{-\infty}^{t} I_{FT}(t-t')\, F(\alpha_L, 0, f_L, t, t') \prod_{m=-\infty}^{\infty} F(\alpha_m, \phi_m, f_m, t, t')\, F(\alpha_m', \phi_m', f_m', t, t')\, dt'\right\} \quad \text{(A3)}$$

where

$$F(\alpha, \phi, f, t, t') = \sum_{k=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} J_k(\alpha) J_{k'}(\alpha) \cdot e^{i(k-k')(2\pi ft+\phi)} e^{i2\pi k' f(t-t')}. \quad \text{(A4)}$$

This result follows from carrying out the integral in the exponent of (A1) and using the identity

$$e^{ia\sin x} = \sum_{k=-\infty}^{\infty} J_k(a)\, e^{ikx}. \quad \text{(A5)}$$

Each term of the integrand of (A3) contains $I_{FT}(t-t')$ and an exponential factor involving $t - t'$, but all other factors are constant; carrying out this integral then leaves (4). The function $\tilde{I}(f)$, used in (4), is the analytic signal of $I_{dc}(V_{dc} + hf/q)$, given by

$$\tilde{I}(f) = 2\int_{0}^{\infty} I_{FT}(t)\, e^{+i2\pi ft}\, dt. \quad \text{(A6)}$$

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Variational Bound Analysis of a Discontinuity in Nonradiative Dielectric Waveguide

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***Abstract* — This paper describes the application of the variational bound method to nonradiative dielectric waveguide for the analysis of scattering by a dielectric obstacle, in this case a rectangular, air-filled discontinuity in the dielectric center strip. Closed-form equations are obtained that can be used directly in the design of networks using reactive components, such as filters. Measured data agree well with the theoretical calculations.**

I. Introduction

The application of specific properties of discontinuities in waveguides forms the basis of a variety of microwave devices. In the nonradiative dielectric waveguide only one such analysis has been reported, by Yoneyama *et al.* [1], where a step discontinuity was described and applied in the design of a filter. Expressions for describing the network are not given.

In this paper, the variational bound (VB) method described by Aronson *et al.* [2] is used to analyze the scattering from a rectangular hole through the dielectric center conductor of the

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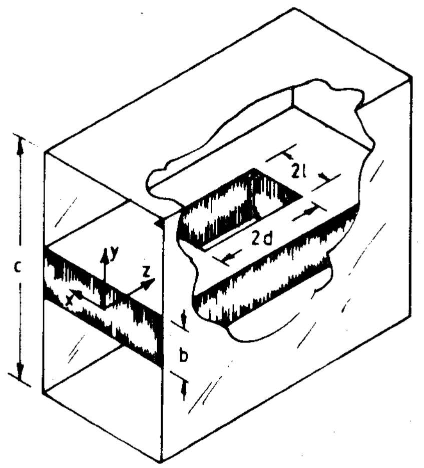

Fig. 1. Isometric view of the rectangular hole in the waveguide center dielectric strip.

NRD guide. A variational bound is obtained on η, the phase shift which measures the amount by which the asymptotic solution for the fields in the guide in the presence of the discontinuity is displaced relative to the guide without a discontinuity. A major advantage of the procedure is that a closed-form expression for the equivalent network is obtained. The design of networks making use of reactive elements is consequently greatly simplified, because it eliminates empirical determination of element values.

II. Analysis of a Square Discontinuity in NRD Waveguide

Because the NRD waveguide is an open structure, it has only a finite number of discrete solutions of Maxwell's equations, while the rest of the eigenvalue and eigenfunction spectrum is continuous. If the structure is boxed, as shown in Fig. 1, the continuous eigenvalue spectrum is discretized, and only discrete solutions to Maxwell's equations are permitted. It is therefore now possible to expand any arbitrary electromagnetic field into the eigenfunctions or waveguide modes, using the orthogonality of the modes.

The walls are, however, placed far enough away so as not to influence the propagation constants of the surface wave solutions, which correspond to the discrete solutions in the open NRD waveguide. Therefore they will not influence the theoretical results obtained.

Fig. 1 shows an isometric view of the guide, and also defines the dimensions. The analysis closely follows that outlined in [2]. In the absence of the obstacle, the odd and even parts of the total electric field are given by

$$\boldsymbol{E}_{Te} = \boldsymbol{e}(x,y)\cos k_z z \tag{1}$$

$$\boldsymbol{E}_{To} = \boldsymbol{e}(x,y)\sin k_z z \tag{2}$$

and $\boldsymbol{e}$ is the form function of the dominant mode, which is a TM_{y11} mode. The total field is obtained as

$$\boldsymbol{E} = \boldsymbol{E}_{Te} + j\boldsymbol{E}_{To}. \tag{3}$$

In the presence of an obstacle the asymptotic forms ($z \to \infty$) of the fields are given by

$$\boldsymbol{E}_e = C_e \boldsymbol{e}(x,y)[-\sin k_z z + \cot \eta_e \cos k_z z] \tag{4}$$

$$\boldsymbol{E}_o = C_o \boldsymbol{e}(x,y)[\cos k_z z + \cot \eta_o \sin k_z z] \tag{5}$$

where η_e and η_o are the even and odd mode phase shifts

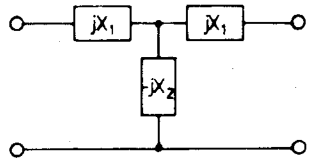

Fig. 2. Equivalent circuit of waveguide discontinuity.

associated with the discontinuity. The equivalent network parameters defined in Fig. 2 are related to η_e and η_o by

$$X_1 = \tan \eta_o \tag{6}$$

$$X_2 = \frac{1}{2}(\tan \eta_e + \cot \eta_o). \tag{7}$$

Using the variational bound method [2], bounds on η_e and η_o may be obtained, from which it is possible to calculate X_1 and X_2. The procedure starts by solving the differential equation

$$\frac{d^2 f}{dz^2} + f(z)\left[k_z^2 - \int_{-l}^{l}\int_{-\frac{1}{2}b}^{\frac{1}{2}b}(\epsilon_r - 1)k_0^2 \boldsymbol{e}\cdot\boldsymbol{e}\,dxdy\right] = 0 \tag{8}$$

where k_z is the propagation constant of the dominant mode, and $\boldsymbol{e}(x,y)$ is the vector form function of the dominant TM_{y11} mode. This is normalized such that

$$\int_S \boldsymbol{e}_t(x,y)\cdot\boldsymbol{e}_t(x,y)\,dxdy = 1 \tag{9}$$

where the area of integration is a cross section of the waveguide, and $\boldsymbol{e}_t$ denotes the transverse part of $\boldsymbol{e}$. The description of the TM_{y11} mode is obtained from [3].

The differential equation (8) reduces to

$$d^2f/dz^2 - \left(V^2 - k_z^2\right)f(z) = 0. \tag{10}$$

In this, V^2 is given by

$$V^2 = \int_{S_g}(\epsilon_r - 1)\boldsymbol{e}(x,y)\cdot\boldsymbol{e}(x,y)\,dS \tag{11}$$

and the surface S_g is defined by $-l < x < l$; $-\frac{1}{2}b < y < \frac{1}{2}b$, and lies in the discontinuity. From (11) it follows that

$$V^2 = (\epsilon_r - 1)\left\{\int_{S_g} E_x^2\,dS + \int_{S_g} E_y^2\,dS + \int_{S_g} E_z^2\,dS\right\}. \tag{12}$$

To satisfy (9), the amplitude of $\boldsymbol{e}(x,y)$ must be

$$\begin{aligned} A = &-\left(\frac{\pi}{a}\cdot\frac{\beta}{\epsilon_r}\right)^2 \cdot \frac{1}{2}a\left[\frac{1}{2}b - \frac{\sin\beta b}{2\beta}\right] - a\left[\frac{\pi}{a}\alpha\frac{\cos\frac{1}{2}\beta b}{\cosh\frac{1}{2}\beta(c-b)}\right]^2 \\ &\cdot\left[-\frac{1}{2}b + \sinh\frac{1}{2}\alpha(c-b)\frac{1}{2}\cosh\frac{1}{2}\alpha(c-b)\right] \\ &-\left[\frac{k^2-\beta^2}{\epsilon_r}\right]^2\cdot\frac{1}{2}a \\ &\cdot\left[\frac{1}{2}b + \frac{\sin\beta b}{2\beta}\right] - a\left[k_0^2 + \alpha^2\right]\cdot\left[\frac{\cos\frac{1}{2}\beta b}{\cosh\frac{1}{2}\alpha(c-b)}\right]^2 \\ &\cdot\left[\frac{1}{2}b + \frac{1}{2\alpha}\sinh\frac{1}{2}\alpha(c-b)\cdot\cosh\frac{1}{2}\alpha(c-b)\right]. \end{aligned} \tag{13}$$

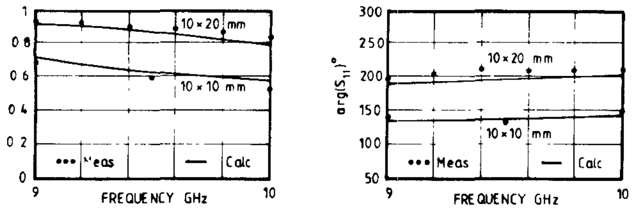

Fig. 3. Measured (dotted lines) and calculated values (solid lines) for the reflection coefficient and phase of the equivalent circuit

A is therefore the necessary normalization constant of $e(x, y)$. The expression for V^2 follows from (12):

$$V^2 = -k_z^2 + \frac{(\epsilon_r - 1) k_0^2}{A} \cdot (k_z \beta)^2 / \epsilon_r \left[l + \frac{\sin 2\pi l/a}{2\pi/a} \right]$$
$$\cdot \left[\frac{1}{2} b - \frac{\sin \beta b}{2\beta} \right] - \left[\frac{k^2 - \beta^2}{\epsilon_r} \right]^2 \cdot \left[l + \frac{\sin 2\pi l/a}{2\pi/a} \right]$$
$$\cdot \left[\frac{1}{2} b + \frac{\sin \beta b}{2\beta} \right] - \left[\frac{\pi}{a} \cdot \frac{\beta}{\epsilon_r} \right]^2$$
$$\cdot \left[l - \frac{\sin 2\pi l/a}{2\pi/a} \right] \cdot \left[\frac{1}{2} b - \frac{\sin \beta b}{2\beta} \right] \Bigg\}. \tag{14}$$

After substitution of A into (14), a parameter K is defined as

$$K^2 = -k_z^2 + V^2.$$

The differential equation (10) now is

$$d^2 f(z)/dz^2 - K^2 f(z) = 0 \tag{15}$$

and the solution to this is

$$f(z) = f_e + f_o \tag{16}$$

where f_e and f_o are the even and odd parts of $f(z)$, and

$$f_e = D_e \cosh Kz \qquad f_o = D_o \sinh Kz. \tag{17}$$

If (3), (4), and (17), together with their derivatives, are equated at the discontinuity boundary, which is a static approximation, then the even and odd mode phase shifts are given by

$$\cot \eta_e = \frac{-\sin k_z d + k_z/K \coth Kd \cos k_z d}{-k_z/K \sin k_z d \coth Kd - \cos k_z d} \tag{18}$$

$$\cot \eta_o = \frac{\cos k_z d + \sin k_z d \, k_z/K \tanh kd}{k_z/K \cos k_z d \tanh Kd - \sin k_z d} \tag{19}$$

where the wavenumber in the z direction is k_z. The transverse wavenumbers, α and β, are solutions of the transcendental equation

$$\beta \tan \frac{1}{2} \beta b = \epsilon_r \alpha \tag{20}$$

subject to the constraint

$$\alpha^2 + \beta^2 = k_0^2 (\epsilon_r - 1) \tag{21}$$

while the guide wavenumber is calculated from

$$k_z = \sqrt{k^2 - (\pi/a)^2 - \beta^2}. \tag{22}$$

It is now possible to calculate the equivalent admittance network for holes of any given dimension using (6) and (7).

III. Experimental Results

The reflection coefficient and phase angle of the equivalent T section of two rectangular holes, of dimensions 10×20 mm and 10×10 mm, respectively, were measured, and the data are plotted in Fig. 3(a) and (b) together with the theoretically calculated values using the equations developed above. The agreement between the various sets of values is good.

IV. Conclusions

A theoretical analysis that makes it possible to calculate the equivalent circuit for a rectangular discontinuity in NRD waveguide has been presented and verified experimentally. A major advantage of the theory is that the equations are in closed form.

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Frequency Normalization of Constant Power Contours for MESFET's

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Abstract —**The constant power contours have been measured on MESFET's with different doping profiles over the frequency range 8–16 GHz for a fixed input power level at different bias points. At each frequency, the contours are normalized with respect to the load for maximum power output; within experimental accuracy, the normalization holds fairly well independent of frequency under certain limits.**

I. Introduction

The design of a power amplifier over a wide frequency range involves, in general, the measurement of constant power contours and constant efficiency contours at different frequencies at some given bias point. The measurement of these contours at different frequencies is very time-consuming. Moreover, one may not have the experimental setup to find the contours at the frequencies of interest. The present investigation shows how these curves can be predicted at other frequencies under certain limits. The concept is an extension of small-signal mismatch circles. In case of large-signal applications, the mismatch contours are no longer circles, because of the way the saturation current and breakdown voltage limits are attained. The contours present some reflection coefficient loci with respect to a given load. The idea was first proposed by Cripps [1]; it has been extended in this paper over a wide frequency range.

In the present case, the contours are normalized with respect to the conjugate of load that gives maximum power with a given input power. The conjugate of optimum load impedance to yield maximum power output may be represented at any given frequency by a parallel combination of R and C. The values of R

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